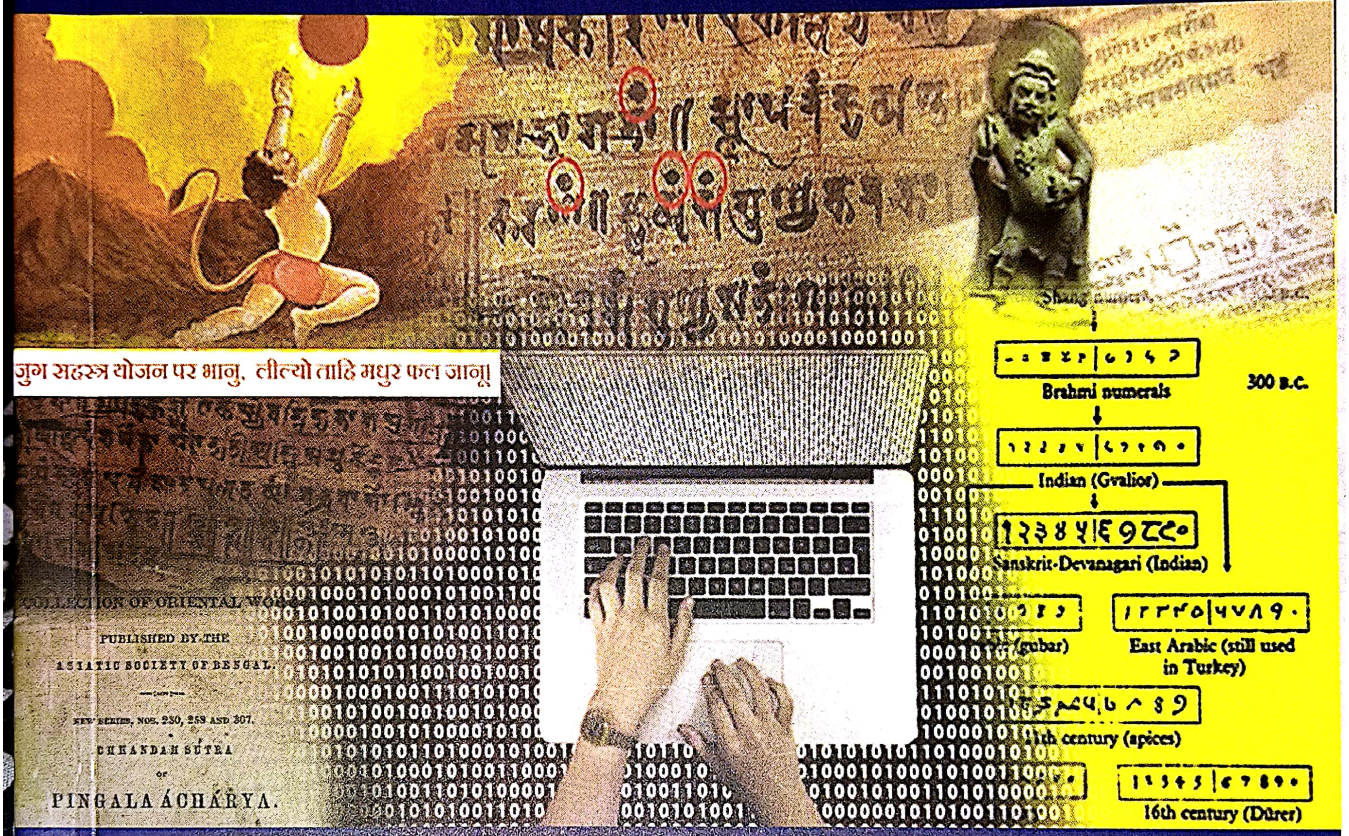
Bharat BoudhIKS

Volume-3

Numerals & Measures

The Bharatiya Perspective

Eternal Guidance : Youth Excellence



A Concise Book for IKS based Competitive Exams (NET and UPSC)

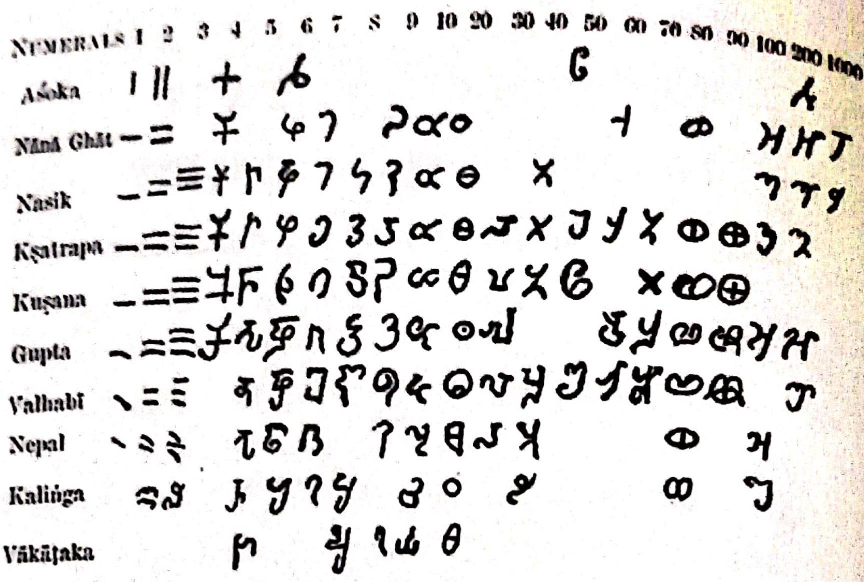
Numerals and Measures: The Bharatiya Perspective

This book explores the remarkable journey of Bhārat's number systems and units of measurement, tracing their roots from ancient Vedic-era registers and inscriptions through to sophisticated methods like the Bhūta-saṃkhyā and Kaṭapayādi systems. You'll discover how early scholars in Bhārat developed the decimal place-value format, invented the concept of shūnya (zero), and represented enormously large quantities-all evidenced in texts like the Yajurveda, Vedāṅga Jyotiṣa, and inscriptions dating back to around 600 CE. These foundational ideas were further developed by luminaries such as Piṅgala (Chandaḥ-śāstra and early binary ideas), Aryabhata, Brahmagupta, and others-blending intellectual rigor with cultural depth. They not only revolutionized trade, astronomy, and science in ancient Bhārat, but also seeded innovations adopted across Asia, the Arabic world, and eventually Europe. This book guides you through these systems in a friendly, grounded way-so you can connect with a living legacy of mathematical insight from Bhārat.

1. Number System

Scientific and technological advancements have greatly benefited from the number system, one of Bharat's most significant contributions to the world. This system was developed in Bharat long before the Common Era (CE), later adopted by Arab scholars in the 8th century CE, and eventually introduced to Western countries by the 11th century CE.

Numbers form the cornerstone of human civilization. The numeral scripts developed to express these numbers are not only symbols of mathematical advancement but also narrate tales of cultural and intellectual evolution. This book delves into the journey of numeral scripts in the Bharatiya subcontinent, tracing their progression from Brahmi numerals to the modern Hindu-Arabic numeral system.



Evolution of Brahmi numerals from the time of Ashoka.
Source: The Hindu-Arabic numerals (published 1911),
Author-Smith, David Eugene, 1860-1944)

1.1 Brahmi Numerals: The Early Form

The Brahmi numeral system is the oldest known numerical notation in the Bharatiya subcontinent, dating back to the Maurya period in the 3rd century BCE. This system was decimal-based but lacked the concepts of zero and positional value.

Examples of Brahmi Numerals

Number	*Brahmi Numeral*
1	·
2	\
3	३
4	३
5	४
6	५
7	ა
8	٦
9	ट
10	९

The evolution of numeral scripts is significant not only from a mathematical perspective but also as a testament to the intellectual achievements of Bharatiya civilization. Innovations like the concept of zero and the positional value system have added new dimensions to global mathematical thought.

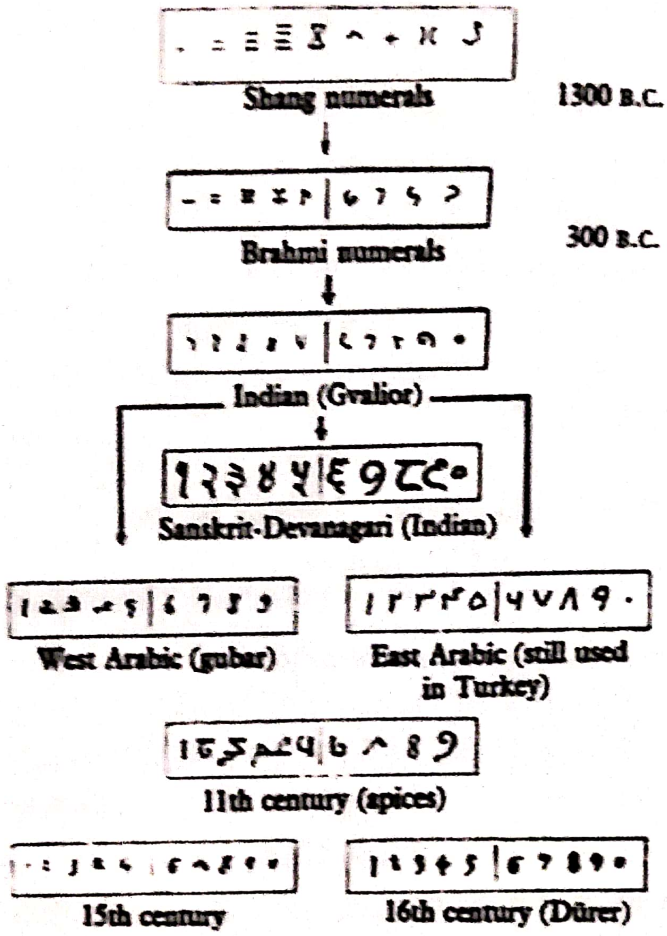


Diagram depicting the evolution of Hindu Arabic numerals.
Source: Karl Menninger (1934) Zahlwort und Ziffer, p. 233, later translated into English (1969) as Number Words and Number Symbols, p. 418. Modified by Frank Swetz (1984) in "The Evolution of Mathematics in Ancient China", in Campbell & Higgins, eds., Mathematics: People, Problems, Results, p. 31)

1.5 Historical Evidence of the Number System in Bharat

Research by Ifrah has provided compelling evidence from both Arab and European sources, confirming that the modern number system originated in Bharat. Drawing on references from 810 CE to 1814 CE, Ifrah highlights observations by past scholars on Bhartiya mathematics. For instance, Laplace praised the Bhartiya numeral system, noting its "ingenious method of expressing every possible number using ten symbols," each with both absolute and place values. He emphasized that this system's simplicity made arithmetic one of the most useful inventions. Similarly, Al-Biruni, in his 1030 CE work on Bharat, noted that while Arabs used letters for calculations based on their numerical values, Bhartiyas did not. He also observed that, just as regional variations exist in the shapes of written letters, the numerical symbols used by Bhartiyas also varied by region

Archaeological evidences further support linguistic findings. In the Indus-Saraswati civilization, for instance, the widths of streets were standardized. Excavations at Kalibangan (Rajasthan) revealed street widths of 1.8 m, 3.6 m, 5.4 m, and 7.2 m, corresponding to standard measurements of 1, 2, 3, and 4 Dhanus, respectively. Similarly, findings from Harappa, Mohenjo-Daro, Dholavira, and Lothal show that construction relied on fired bricks of precise geometric proportions, with standard ratios of length, width, and depth (4:2:1).

The *Arthaśāstra* mentions two types of Dhanus for measuring lengths and distances:

- Dhanus = 96 Aṅgulas
- Gārhapataya Dhanus = 108 Aṅgulas

Additionally, a legal document from 594 CE found in Bharukachcha (Broach), Gujarat, features numbers written in the place-value format that is still used today. An inscription from Gwalior, dated 876 CE (Samvat 933 in the Vikrama Calendar), records the numbers 50 and 270 with a small circle placed appropriately to represent zero. Another inscription, dating from the ninth century CE, also demonstrates the use of zero. In 1881, the discovery of the *Bhakshali Manuscript*, a scroll that contains one of the earliest recorded uses of zero, was carbon-dated to the third or fourth century CE, further corroborating the Bhartiya origins of the numeral system.

2. Bhartiya Numeral System – Key Features

The origins of the Bhartiya numeral system can be traced back to the Vedic period, where Sanskrit had unique names for numbers:

- The first nine digits are: *ekam, dve, trīṇi, catvāri, pañca, ṣaṭ, sapta, aṣṭa, nava.*
- Numbers from 10 to 100 in steps of ten are: *daśa, viṃśati, triṃśat, catvāriṃśat, pañcāśat, ṣaṣṭi, saptati, aśīti, navati, śata.*

The Bhartiya numeral system made significant contributions to scientific progress:

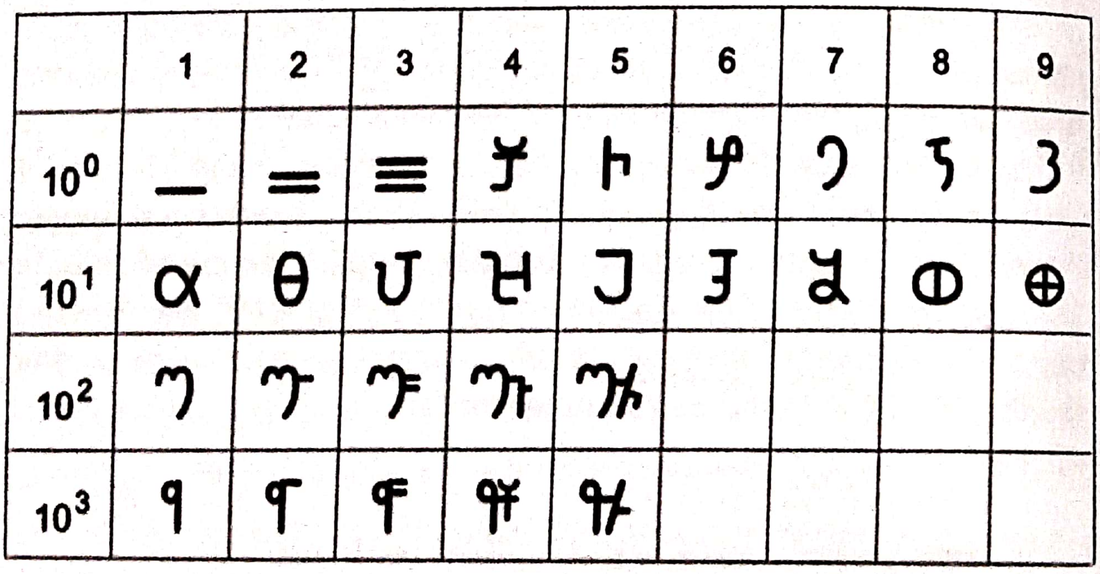
- A legacy of large number names, which made it easier to understand and work with large quantities.

- The development of the place-value system, a major advancement for representing numerals.
- The concept of zero, which transcended its role as a mere placeholder.
- A decimal system that revolutionized arithmetic operations.

3. The Concept and Importance of Zero

The concept of zero emerged around 500-300 BCE and was fully developed in Bharat by 600 CE. The Sanskrit word *Śūnya* denoted zero, and its use as a numeral, beyond being a mere placeholder, had a profound impact on mathematics. Zero allowed for complex calculations, such as calculus, and was fundamental to the development of binary arithmetic used in modern computers.

The concept of zero as a number with its own properties was explored in *Bīja-gaṇita* by Bhāskara II, who detailed its behavior in mathematical operations like addition and subtraction. Zero's properties were such that its value did not change when added to or subtracted from other numbers.



Brahmi numerals signs of the 2nd century CE.
Source: Stephen Chrisomalis (Q59611138), Numerical notation: A Comparative History, Cambridge [et al.]: Cambridge University Press, 2010, ISBN 978-0-521-87818-0, p. 189, and Richard Salomon, Indian Epigraphy, New York/Oxford: Oxford University Press, 1998, ISBN 0-19-509984-2, p. 58)

In addition to mathematical use, the term *Śūnya* was first introduced by the Bhartiya philosopher Piṅgala in the second century BCE in his work *Chandaḥśāstra*, which focused on Sanskrit poetry meters. There, *Śūnya* referred to an absence of quantity, laying the groundwork for its later adoption in mathematics. Brahmagupta further advanced the use of zero as a distinct numeral in 628 CE, providing a formalized symbol for it.

Ancient Bhartiya sages made significant contributions in developing the idea of zero, transforming it from a mere placeholder to a number with its own distinct properties. Below, we delve deeper into how the concept of zero emerged in Bharat, its philosophical roots, and its long-lasting impact on mathematics and science.

3.1 Zero in Ancient Bhartiya Thought

The term **Śūnya** in Sanskrit originally referred to a void or the absence of quantity, but over time, its meaning evolved into the concept of zero as a number. The earliest mention of zero can be traced back to **Piṅgala**, the ancient Bhartiya scholar, in his work *Chandaḥśāstra* (2nd century BCE). Piṅgala used the term **Śūnya** to represent the absence of a "beat" in the context of Sanskrit poetry, where each syllable could either be "short" or "long." When there was no syllable, Piṅgala referred to it as Śūnya, marking the beginning of zero's journey in mathematics.

3.2 Zero in Brahmagupta's Work

The concept of zero was fully formalized in **628 CE** by the mathematician **Brahmagupta** in his seminal text, *Brahmasphuṭasiddhānta*. Brahmagupta was the first to define zero as both a number and a symbol in its own right. He described it as the "result of subtracting a number from itself," leading to zero, and provided rules for its use in arithmetic.

"रुणं रुणसंख्याया, कंचित् स्वाधिकया स्वात् समानाय च शून्यं।"

In Brahmagupta's work, the following rules for zero were established:

- Zero plus a number equals the number itself (i.e., $0 + a = a$).
- Zero minus a number equals the negative of that number (i.e., $0 - a = -a$).
- A number minus zero equals the number (i.e., $a - 0 = a$).
- Zero multiplied by any number equals zero (i.e., $0 \times a = 0$).
- Division by zero was deemed undefined, marking an early attempt to recognize its paradoxical nature (i.e., $a \div 0$ = undefined).

Brahmagupta's rules were groundbreaking as they laid the foundation for modern arithmetic operations involving zero, transforming it from a mere placeholder to a fully functional number.

3.3 Zero and the Decimal System

The full significance of zero became apparent when it was used in combination with the **decimal system**. In the **place value system**, the position of a numeral dictates its value, and zero played an essential role in this system by distinguishing between, for example, 205 and 25. This system allowed for the representation of large numbers and calculations, providing a powerful tool for mathematical and astronomical analysis. This system was integral to the work of later mathematicians, such as **Aryabhata**, who used it in his astronomical calculations. The Bhartiya decimal system was eventually transmitted to the Islamic world and later to Europe, forming the basis of the modern number system.

The earliest known use of zero in the decimal system is seen in the **Inscription of Gwalior** (around 876 CE), where the zero symbol (a small dot) is clearly visible in a number. This represents the first definitive instance of zero as a place value.

World's Oldest Recorded Zero at the Chaturbhuj Temple in Gwalior

Source: Encryptedpast.com

3.4 Bījagaṇita and Bhāskara-II Contributions

Another significant Bhartiya mathematician, **Bhāskara-II** (12th century CE), also known as **Bhaskaracharya**, made profound contributions in his work *Bījagaṇita* (The Book of Algebra). Bhāskara II extensively discussed the concept of zero and algebraic equations, delving into its use in **indeterminate equations** (those with no solution or multiple solutions) and **fractions**.

In this work, Bhāskara II formulated concepts such as:

- Zero as an additive identity (i.e., a + 0 = a).
- Zero as a multiplicative identity (i.e., 0 × a = 0).
- Mathematical operations involving negative numbers, which were often conceptually linked to zero in algebraic expressions.

One of Bhāskara II's famous contributions was to solve quadratic equations and work with what we now call **differential calculus**, which required a sophisticated understanding of zero, particularly in limits and infinitesimals.

3.5 Zero's Philosophical and Spiritual Dimensions

The concept of zero had a profound influence not just on mathematics, but also on **Bhartiya philosophy**. In **Hinduism, Śūnya** is seen as representing the **ultimate reality** or the void from which all creation emanates. The **Upanishads**, which are ancient texts of Bhartiya philosophy, discuss the **nature of reality** as being rooted in the "void," and zero as representing the essential oneness of existence.

Similarly, in **Buddhism**, the concept of **Śūnyatā** (emptiness or voidness) describes the idea that all phenomena are devoid of inherent existence and that all things are interdependent. Zero, in this philosophical context, symbolizes the absence of inherent, independent existence and the transient nature of reality.

3.6 Zero in the Modern Era

The contributions of Bhartiya mathematicians, particularly Brahmagupta and Bhāskara-II, laid the groundwork for modern mathematics. In contemporary mathematics and science, zero is essential in a wide range of disciplines, from **algebra** to **calculus** and **computing**.

In modern **computer science**, for example, the binary numeral system, which underpins all modern digital technology, is based entirely on the concept of zero and one. Every calculation, every algorithm, and even the storage of data in computers can be traced back to the ancient Bhartiya understanding of zero.

3.7 Shlokas and Textual References to Zero

The development of zero is also deeply embedded in the spiritual and philosophical teachings of ancient Bhartiya texts. For example:

- In the **Bhagavad Gita**, Lord Krishna speaks about the eternal nature of the soul, describing its existence as "formless" or "Śūnya," where he says, "The soul is neither born, nor does it die" (Bhagavad Gita 2.20). This idea of formlessness is philosophically aligned with the concept of zero, signifying something that transcends physical existence.
- In **Piṅgala's Chandaḥśāstra**, the concept of Śūnya is used to represent the absence of a beat, laying the philosophical groundwork for zero as the absence of quantity, a concept later formalized in mathematics.
- **Brahmasphuṭasiddhānta (Brahmagupta, 628 CE): "शून्यं प्रतिशून्यं च संज्ञायाम्।"** – Brahmagupta defines the concept of zero as a number, providing the rules for its arithmetic operations.
- **Chandaḥśāstra (Piṅgala, 2nd century BCE): "शून्यं तु निस्तरङ्गं हृदयाद्विजिनः प्रवृत्तम्।"** – Piṅgala's reference to Śūnya as representing the absence of a beat, an essential concept for understanding nothingness.

Zero is not only a mathematical tool but also a profound philosophical symbol, reflecting both the absence and the potential for creation, a duality that continues to inspire thought across many domains of knowledge.

4. Representation of Large Numbers

The ancient Bhartiyas were well-versed in handling large numbers, as demonstrated by references in the *Chāndogya Upanishad*, which discusses the idea of infinite numbers. The *Ṛigveda* contains names for large numbers, many of which are not simple multiples of ten. The *Taittirīya-saṃhitā* refers to numbers as large as $10^{13,}$ and other texts, such as the Bṛhadāraṇyaka-Upanishad, mention numbers up to 1014. These large numbers were important for astronomical calculations, which often dealt with vast quantities.

The naming of large numbers in Sanskrit was systematic and followed three main principles:

1. Unique names were given to numbers from 0 to 9 (e.g., śūnya, ekam, dve, etc.).
2. Additive principles were used for numbers between 11 and 99. For example, 12 was named dve-daśa (2 + 10), and 84 was named catvāri-aśīti (4 + 80). A subtractive principle was also applied, such as 29 being named ekona-triṃśat (30 - 1).
3. **Multiplicative principles** were employed for numbers of higher powers of 10. For instance, 7000 was called *sapta-sahasram* (7 × 1000), and 80,000 was called *aṣṭa-ayuta* (8 × 10,000).

In summary, the Bhartiya numeral system, with its advanced use of zero and place-value notation, laid the foundation for much of modern mathematics and computation. The systematic approach to naming and handling large numbers further demonstrates the sophistication of ancient Bhartiya mathematics, which continues to influence scientific progress today.

5. Place Value of Numbers in Ancient Bhartiya Mathematics

The concept of place value in numbers, a foundational element of modern arithmetic, was fully developed in ancient Bharat. The *Agni Purāṇa* explains this principle by stating: "*...in the case of multiples from the units place, the value of each place is ten times the value of the preceding place...*" This clearly illustrates the place value system, where the value of digits changes depending on their position in a number.

Similarly, the *Vāyu Purāṇa* provides another reference to this system: "*..from one place to the next in succession, the places are in multiples of ten. The eighteenth place is called parārdha...*" This not only emphasizes the decimal nature of the place-value system but also suggests the conceptualization of extremely large numbers, extending the system to at least 18 places.

In the *Śārīraka-bhāṣya*, a commentary by Śaṃkarācārya, a passage highlights the idea of place value with an analogy: "*An individual by the name of Devadutta may be called differently as a father, son, son-in-law, brother, grandson, child, youth, etc., just as although the stroke is the same, yet by a change of place it acquires values-one, ten, hundred, thousand, etc.*" This comparison effectively illustrates the shift in value based on position, just as a name can represent different roles depending on context.

A similar explanation of place value appears in *Patañjali's Yoga-sūtra*, further emphasizing its significance in ancient Bhartiya thought. Additionally, in the mathematical text *Gaṇita-sāra-saṅgraha*, written around 850 CE by Mahāvīrācārya, an example of place value is provided through a numerical operation. He describes the number "12345654321," which is the square of "111111," showing how numbers are manipulated using the place value system. This example not only illustrates the place value principle but also demonstrates the mathematical creativity of ancient Bhartiya scholars. These references collectively attest to the advanced understanding of place value and numerical operations in ancient Bharat, which laid the groundwork for the modern number system.

6. The Decimal System

In the opening verses of the *Līlāvatī*, the renowned Bhartiya mathematician Bhāskarācārya (Bhaskara I) acknowledges that his ancestors had developed a place-value system based on multiples of ten, which is essentially the decimal system. This early use of the decimal system in Bharat predates its adoption in the Western world by many centuries. The decimal number system, with its base-10 structure, was fully conceptualized and used in Bharat long before it spread to other cultures.

This system's place-value nature, where the value of a digit depends on its position within a number (ones, tens, hundreds, and so on), made it significantly more efficient than earlier numeral systems. The development of this place-value system in Bharat laid the foundation for the modern arithmetic that we use today, demonstrating the advanced mathematical thinking of ancient Bhartiya scholars.

7. Representing Numbers Using Unique Approaches in Ancient Bhartiya Mathematics

Ancient Bhartiya mathematics integrated numerical concepts with literature and poetry in a way that was both creative and practical. To achieve this, two distinct systems were commonly used for representing numbers:

1. Bhūta-saṃkhyā System *(Source: Datta, B., & Singh, A. N. (1935). History of Hindu Mathematics (Vol. 1 & 2). Calcutta University Press.)*
2. Kaṭapayādi System *(Source: Katz, V. J. (2007). A History of Mathematics: An Introduction (3rd ed.). Pearson and Filliozat, J. (2004). The Classical Doctrine of Indian Medicine: Its Origins and Its Greek Parallels. Munshiram Manoharlal Publishers)*

7.1 Bhūta-saṃkhyā System

The *Bhūta-saṃkhyā* system expresses numbers through words that represent various familiar entities, concepts, or objects from daily life, mythology, or nature. These words, which could be physical objects, animals, body parts, or even deities, are used to signify specific numbers. The beauty of this system lies in its flexibility and connection to cultural and philosophical ideas.

For example, in the *Bhūta-saṃkhyā* system:

- The word *chandra* (meaning "moon") represents the number *one*.
- The word *eye* signifies the number *two*.
- The word *guṇa* refers to the number *three*.

Other synonyms for *moon*, such as *śaśin*, *vidhu*, *soma*, and *indu*, also correspond to the number one. The system is rich in symbolism, drawing from natural phenomena and spiritual ideas. Words can even represent fractions, such as *kalā* for 1/16, *kuṣṭha* for 1/12, and *śapha* for 1/4.

While this system offers flexibility and poetic richness, it requires users to be familiar with synonyms and metaphorical meanings in order to correctly interpret the numbers. The system is not strictly formalized; rather, it is open-ended, allowing for artistic expression and a blend of literature with mathematics.

Common Sources of Words in Bhūta-saṃkhyā System

The *Bhūta-saṃkhyā* system draws from several categories of words:

1. **Numerical Terms:** Words like *śūnya* (zero), *ekam* (one), *dve* (two), *trīṇi* (three), *catvāri* (four), *pañca* (five), *ṣaṭ* (six), *sapta* (seven), *aṣṭa* (eight), *nava* (nine). *(Source: Hayashi, T. (2003). Indian Mathematics. In Helaine Selin (Ed.), Encyclopaedia of the History of Science, Technology, and Medicine in Non-Western Cultures. Springer.)*
2. **Physical Entities:** Natural elements such as *earth (1)*, *moon (1)*, *stars (∞)*, *mountains (7)*, *fire*, *sky (1)*, and *directions (4/10)* are used to represent numbers. *(Source: Katz, V. J. (2007))*
3. **Animals:** Symbols of animals such as the elephant (with 8 tusks), the serpent (based on 5 or 7 hoods), and the horse (with 4 legs) were also used in the representation of numbers. *(Source: Pingree, D. (2003). The Logic of Non-Western Science: Mathematical Traditions of South Asia. ISHM.)*
4. **Parts of the Body:** Bodily parts such as the eyes (2), hands (2), nostrils (2), and the seven bodily elements (7) were also used in representing numbers in the bhūta-saṅkhyā system. *(Source: Joseph, G. G. (2011). The Crest of the Peacock: Non-European Roots of Mathematics (3rd ed.). Princeton University Press.).*
5. **Deities:** Deities were also used to convey numerical values, such as Brahmā (1), the Trimūrti—Brahmā, Viṣṇu, Maheśa (3), the Pañcadeva—Gaṇeśa, Viṣṇu, Śiva, Durgā, Sūrya (5), and the twelve Ādityas (12), and so on. *(Source: Hayashi, T. (2003))*
6. **Natural and Temporal Concepts:** Temporal and natural elements were also used in this system, such as the seasons (6), months (12), days of the week (7), and the five great elements—earth, water, fire, air, and space (5). *(Source: Datta, B., & Singh, A. N. (1935).*

Flexibility and Aesthetic Integration

The *Bhūta-saṃkhyā* system offers an open-ended list of representations. It is not governed by rigid rules but is rather shaped by the user's creativity and knowledge. This allows mathematicians, poets, and scholars to integrate numerical representation with the aesthetic beauty of language, enabling a poetic and culturally rich approach to mathematics. This system exemplifies how ancient Bhartiya mathematics was not just a technical field but also an art form, where the boundaries between math, literature, and philosophy were fluid, making numbers not only tools for calculation but also carriers of meaning, beauty, and culture.

7.2 Kaṭapayādi System: A Unique Method of Representing Numbers

The **Kaṭapayādi System** is another ingenious approach developed in ancient Bharat to represent numbers using the alphabet. The earliest known use of the Katapayadi system is found to have been made in the 7th century AD by mathematicians of Kerala such as Vararuchi, and later by astrologers of the Kerala School. This system offers a clever method of encoding numbers into words. Unlike the **Bhūta-saṃkhyā system**, which uses common entities and metaphors to signify numbers, the *Kaṭapayādi* system assigns specific numerals to each consonant. The result is a system that combines the structure of language with numerical representation, often used for mnemonic purposes, especially in ancient texts, poetry, and scholarly works.

(Source: Pingree, D. (1981). Jyotihśāstra: Astral and Mathematical Literature. Otto Harrassowitz.)

7.2.1 How the Kaṭapayādi System Works

In this system, each consonant is assigned a number from 0 to 9. The vowels, however, do not have direct numerical values and are typically used as place holders or to separate numbers. By combining consonants, one can form words, and by decoding these words letter by letter, the corresponding number can be revealed.

Some of the key rules governing the Kaṭapayādi system are as follows:

1. **Vowels Indicate Zero:** In this system, vowels that stand alone signify the number **zero**. This rule helps distinguish numbers from non-numerical words and maintains clarity in the decoding process.

2. **Consonants Represent Numbers:** Each consonant is associated with a specific number between 0 and 9. The system allows for some flexibility, with multiple consonants being assigned to each number. Here is an example of the common assignments:

Kaṭapayādi System Table

Digit	***Letters (Transliteration)***
1	क (ka), ट (ṭa), प (pa), य (ya)
2	ख (kha), ठ (ṭha), फ (pha), र (ra)
3	ग (ga), ड (ḍa), ब (ba), ल (la)
4	घ (gha), ढ (ḍha), भ (bha), व (va)
5	ङ (ṅa), ण (ṇa), म (ma), श (śa)
6	च (ca), त (ta),ष (ṣa)
7	छ (cha), थ (tha),स (sa)
8	ज (ja), द (da), ह (ha)
9	झ (jha), ध (dha)
0	ञ (ña), न (na)

Notes

- **Vowels (a, ā, i, ī, etc.) are not assigned values**; they are used for pronunciation.
- Multiple consonants can share the same value.
- **Digits are interpreted in reverse order** of the letters used. For example:
- Word: "**naga**" (नग) → ग = 3, न = 0 → value = **03**
- Word: "**kapi**" (कपि) → प = 1, क = 1 → value = **11**

3. **Terminal Consonants:** When consonants are used in conjunction with each other, the terminal consonant (the one preceding a vowel) is considered the key letter for identifying the corresponding number. This rule ensures that the value of the word is derived from the last consonant before a vowel, simplifying the decoding process.
4. **Standalone Consonants:** If a consonant appears without being followed by a vowel, it does not contribute to the numeric value. Therefore, such consonants are ignored in the number-decoding process.
5. **Reading the Numbers:** Similar to the **Bhūta-saṃkhyā system**, numbers encoded in the Kaṭapayādi system are to be read as units, tens, hundreds, and so on, in the standard place-value order. For

example, if the word for a number is constructed by using consonants corresponding to *6* (for *cha*) in the hundreds place, *5* (for *ma*) in the tens place, and *8* (for *da*) in the ones place, the number represented would be **658**.

7.2.2 Applications and Uses

The Kaṭapayādi system was widely used in Sanskrit literature, especially in poetic and mnemonic contexts. It enabled scholars to encode large numbers or intricate mathematical concepts within literary works, making it easier to remember them or hide them in plain sight for esoteric purposes. For example, the famous text **"Chandahśāstra"** by Piṅgala, which deals with the structure of Sanskrit poetry, utilizes the Kaṭapayādi system to represent certain large numbers and mathematical formulas.

The system's ability to blend numbers with letters also made it an ideal tool for composing texts where both linguistic and mathematical elements needed to be preserved or remembered. In such texts, scholars could encode complex mathematical or astronomical calculations in the form of mnemonic verses.

7.2.3 Example of the Kaṭapayādi System

To illustrate the system, let's decode a word using the Kaṭapayādi rules. Suppose we have the word *"rāma"*:

- The consonant *r* corresponds to 2.
- The vowel *a* indicates a placeholder, so it doesn't affect the number.
- The consonant *m* corresponds to 5.
- The vowel *a* again indicates a placeholder.

Thus, the number represented by the word *"rāma"* would be **52**.

7.2.4 Advantages and Flexibility

One of the major advantages of the Kaṭapayādi system is its flexibility in encoding large numbers or even more complex calculations. It allowed for the combination of language and numerics in a way that was easily remembered and was deeply integrated into the cultural context of ancient Bharat. Moreover, this system was not confined strictly to mathematical texts but also found use in literature, where it could serve both as a code and as an artistic device.

The Kaṭapayādi system, along with the *Bhūta-saṃkhyā* system, exemplifies the advanced mathematical thought and creative genius of ancient Bhartiya scholars, showing how mathematics was seamlessly integrated with language and culture. These systems highlight the intellectual and artistic contributions of Bharat to the development of mathematical ideas and methods, many of which have had a lasting influence on the world.

8. Ancient Bhartiya Measurements for Time, Distance, and Weight

Ancient Bhartiya civilization had a well-established system of physical measurements for time, distance, and weight, which were used extensively in trade, commerce, and scientific thinking. These measurements, mentioned in various ancient texts, reflect the advanced understanding of quantities and their applications. Notable scholars like **Bhāskarācārya** (Bhaskara-I) and works like the *Arthaśāstra* and *Līlāvatī* provide detailed accounts of these systems.

9. Ancient Indian Measurement System

In ancient India, a highly developed and organized system was created for measuring time, distance, and weight. This system was not only rich from a scientific and mathematical perspective but was also deeply integrated into trade, commerce, Ayurveda, and daily life. References to these measurements are found in various ancient texts, which reveal the profound scientific outlook of the society of that time.

9.1 Units Used in Physical Measurements

The renowned mathematician Bhāskarācārya (Bhāskara I) mentions various units of length, time, and mass measurement in his treatise Līlāvatī. These units were used in many fields, including trade, commerce, and astronomy. For instance, the Līlāvatī describes the smallest unit such as trasareṇu (a minute unit composed of three atoms), indicating that the scholars of that time had developed the concept of atoms and extremely fine measurements.

9.2 Measurements and Standards in the Arthaśāstra

In Books II, Chapters 19 and 20 of the Arthaśāstra authored by Kauṭilya (Chanakya), a detailed description of the Mauryan period measurement system is provided. It lists standardized units of length, time, and weight, which were used in trade and tax assessment. For example, the daṇḍa (approximately 6 feet) was a unit of length, while units like pala and tola were commonly used for measuring weight.

9.3 The Role of Measurement in Ayurveda

Ancient Ayurvedic texts describe in great detail the precise quantities of substances used in the preparation of medicines. For instance, texts like the Charaka Saṃhitā and Suśruta Saṃhitā mention units such as rasa, prastha, māna, karṣa (approximately 12 grams), and ratti (approximately 0.12 grams) for measuring liquids and solids. The concept of the atom was also significant here, as it enabled precise measurement and ensured the purity and effectiveness of medicines.

9.4. Importance of Measurement in Science and Daily Life

Ancient measurement systems were not limited to scientific knowledge alone; they were also applied in daily life. Units such as ghaṭikā, pala, and nimeṣa were used for measuring time, which played an essential role in astronomical calculations and the preparation of calendars (pañcāṅga). This clearly illustrates the deep harmony between practical science and social life in ancient India.

10. Scientific Thought in Bhartiya Shastras: A Perspective on Measurement

10.1 Time Measurements

In ancient India, time was perceived not only as a practical aspect of life but also as a cosmological concept. The measurement of time was both philosophical and astronomical, with units ranging from small fractions of the day to the vast expanses of cosmic cycles.

10.1.1 Kalpa (Cosmic Day)

- **Definition**: A Kalpa is a day of Brahma (the Creator), lasting 4.32 billion years. This unit reflects the cyclical nature of the universe in Hindu cosmology.

  Reference: *Mahābhārata*, Śānti Parva, 358.56:

  "सप्तपदं महात्मनं कालं ब्रह्मा स्वयम्भुवा।"

Translation: "The seven-petaled great soul, the time (Kalpa) of Brahma, self-born."

This verse emphasizes the vastness of time in the cosmic cycle, aligning with the concept of Kalpa.

Time Measurement Units

S.No.	*Unit*	*Description*
1	Truṭi	The smallest unit of time — approximately 1/33,750 second
2	Vedha	100 Truti = 1 Vedha
3	Lava	3 Vedha = 1 Lava
4	Nimeṣa	3 Lava = 1 Nimeṣa
5	Kṣaṇa	5 Nimeṣa = 1 Kṣaṇa
6	Kālā	1/30 of a Muhūrta (≈ 96 seconds)
7	Nāḍī (i.e., 60 Nāḍī × 24 min = 1440 min = 24 hrs)	Approximately 24 minutes
8	Muhūrta	Approximately 48 minutes = 1 Muhūrta

Sources: Vishnu Purana, Surya Siddhanta, Brahmanda Purana, Manusmriti, Kim Plofker – Mathematics in India, Time Measurement in Ancient India – Balagangadhara Rao.

10.1.2 Muhurta

- **Definition:** A Muhurta is 1/30th of a day, approximately 48 minutes. It is used in rituals to determine auspicious timings.

  Reference: *Rigveda*, 10.191.4:

  "मूर्तिः प्रीतिः सुतं व्रजेयुः मुहूर्ते संप्रजापते।"

Translation: "The form, the delight, the son, let them go in the Muhurta, O Prajāpati."

This verse indicates the use of Muhurta in rituals, highlighting its importance in Vedic traditions.

10.1.3 Aryabhata's Time Calculations

- **Insight:** Aryabhata calculated the sidereal day and solar year, showcasing a profound understanding of celestial movements.

  Reference: *Aryabhatiya*, Chapter 2, Verse 6:

  "अर्धमासो दशगुणं तु यत्र सूर्येण समं यमः।"

Translation: "The half-month is ten times that in which the Sun moves the same distance."

This verse reflects Aryabhata's method of calculating time based on the Sun's movement.

11. Measures of Length, Volume, and Mass

Ancient India's measurements for length, volume, and mass were closely tied to human anatomy and the environment, with units such as the Angula, Hasta, and Vitasti used widely in trade, construction, and daily life.

11.1 Length Measurements

- **Angula**: The width of a human finger, used in early measurements, especially in architecture and sculpture.

  Reference: *Vishwakarma Samhita*:

  "अङ्गुलं रूपं जपेत्।"

  Translation: "Let the form be measured by the finger."

  This verse emphasizes the use of the Angula in architectural measurements.

- **Hasta**: The length of a person's arm, from the elbow to the tip of the middle finger, approximately 18 inches (45 cm). This was a commonly used unit for measuring both length and breadth.

  Reference: *Vastu Shastra*:

  "हस्तस्य एकं यथार्थं चत्वारि"

  Translation: "One Hasta is the actual measure, four times."

  This verse discusses the use of Hasta in building design proportions.

- **Yojana**: A larger unit of measurement used for longer distances, like the stretch of land or the distance between cities or towns.

  Reference: *Hanuman Chalisa*, Verse 18:

  "जुग सहस्त्र योजन पर भानु, लील्यो ताहि मधुर फल जानू।"

  Translation: "The Sun is at a distance of thousands of Yojanas, thinking it to be a sweet fruit."

  This verse metaphorically describes the vast distance to the Sun, using the unit, Yojana.

  Meaning: According to the *Hanuman Chalisa*, Hanuman is described as having leapt toward the Sun, mistaking it for a ripe fruit. The text uses the phrase **"Yuga sahasra yojana"** to describe the distance he covered:

- "Yuga" = 12,000 years
- "Sahasra" = 1,000
- "Yojana" = An ancient unit of distance (commonly taken as ~12.8 km)

So, the calculation becomes:

12,000 × 1,000 × 12.8 km = 153,600,000 km (or 153.6 million km)

This is remarkably close to the modern scientific estimate of the average distance from Earth to the Sun, which is about **149.6 million km.**

11.2 Weight and Volume Measurements

There were many weight measurements units followed in Ancient Bharat as given in the table.

Weight Measurement Units		
Unit	***Approx. Weight***	***Description***
Ratti (Raktikā)	~0.1215 g	Based on the weight of a Gunja seed; widely used in jewelry
Māṣa	8 Ratti ≈ 0.97 g	Common in trade and medicine
Kārṣa	16 Māṣa ≈ 12 g	Ayurveda & medicine
Suvarṇa	16 Māṣa (same as Kārṣa)	Also a standard for gold coins
Tola	12 Māṣa ≈ 11.66 g	Still used in India for gold and silver measurement
Pala	4 Tola ≈ 46.6 g	Common in Ayurveda and trade
Prasṛti	~80 g	Ayurveda measurement unit
Prastha	12 Pala ≈ 560 g	Used for grains and liquids
Śarāva	~1.5 kg	Higher food and commodity measure

- **Kupa:** Used to measure liquids and grains, approximately 4.5 liters.
- **Ghati:** A unit of volume, used for grains and other dry goods, about 0.9 liters.
- **Pala:** A unit used to measure grains and metals, weighing around 320 grams.
- **Reference:** *Arthaśāstra*, 2.16:

  "पालं पञ्चतिमूलं चा।"

  Translation: "The Pala is the basic unit."

This verse refers to Pala as a unit of mass for measuring agricultural produce and goods.

(Source: Srinivasan, S. (1979). Mensuration in Ancient India. Delhi: Ajanta Publications.)

Impact on Trade, Commerce, and Science

The standardization of measurements for time, length, and weight was crucial for the functioning of ancient Bhartiya society, particularly in trade and commerce. The system ensured uniformity, making it easier to engage in commercial transactions across vast distances. Additionally, the precise measurement of time, distance, and weight played an essential role in scientific endeavors such as astronomy, medicine, and metallurgy.

In conclusion, ancient Bhartiya measurement systems, especially the concept of **paramāņu**, highlight the advanced mathematical and scientific knowledge possessed by ancient scholars. These systems were integral to a variety of fields, including commerce, trade, **Āyurveda**, and astronomy, showcasing the depth of Bharat's intellectual heritage in quantifying and understanding the natural world.

12. Piṅgala and the Binary System

Piṅgala, a pioneering scholar who lived between 200–300 BCE, made significant contributions to both the fields of **literature** and **mathematics**. His work, particularly the *Chandaḥ-śāstra* (the treatise on **prosody**), laid the groundwork for several fundamental mathematical concepts, including **combinatorial mathematics**, the **binary system**, and the formalization of the concept of **zero** or *śūnya*. His mathematical insights were ahead of their time, influencing later developments in both **mathematics** and **linguistics**.

12.1 Piṅgala's Chandaḥ-śāstra and Mathematical Insights

In the *Chandaḥ-śāstra*, Piṅgala primarily focused on the rules of **prosody**, which is the study of metrical patterns in poetry. He introduced a system to categorize syllables into two types:

1. **Laghu (Short Syllable):** This is any syllable with a short vowel sound. For example, a syllable like "ka" (where "a" is a short vowel) would be classified as Laghu.
2. **Guru (Long Syllable):** This type of syllable can occur in several forms:
   - A syllable with a long vowel (e.g., "kā" where "ā" is a long vowel).

- A short syllable followed by a conjunction of consonants (e.g., "kr" or "sk").
- A short syllable followed by the nasal *ṃ* (anusvāra) or the voiceless aspiration mark *visarga* (denoted by ":").
- The final syllable of a *quarter meter* (an optional case depending on the meter in use).

These two types of syllables, **Laghu** and **Guru**, are analogous to **short** and **long** in the context of sound length, and were critical for the study of the structure of Sanskrit poetic meters.

12.1.1 Mathematical Concepts in Chandaḥ-śāstra

Beyond his work in poetry, Piṅgala introduced several mathematical ideas that were groundbreaking for his time. His **binary system** approach laid the foundations for understanding how combinations of *Laghu* and *Guru* syllables can generate different metrical patterns, which eventually influenced **combinatorics** and the development of the **binary number system**.

12.1.2 Binary System and Piṅgala's Influence

Piṅgala is often credited with the earliest known application of binary numbers, which is central to modern computing. In the *Chandaḥ-śāstra*, he mapped the combinations of the two syllables (Laghu and Guru) to a binary system of *0* and *1*. Here, the **Laghu syllable** could be represented as **0** (short) and the **Guru syllable** as **1** (long). This binary representation is structurally similar to modern binary code, where every number is expressed as a series of 0s and 1s.

Piṅgala's exploration of binary patterns helped in formulating the **concept of combinations** and **permutations** for strings of syllables, which is closely linked to **combinatorics**, a branch of mathematics that deals with counting, arrangement, and combination of elements.

12.2 De Bruijn Sequence

One of Piṅgala's contributions is essentially an early version of the **De Bruijn sequence**, a concept in combinatorial mathematics that involves generating sequences where every possible string of a given length occurs exactly once as a substring. While Piṅgala did not have the modern framework of combinatorics, his work with the Laghu and Guru syllables is seen as a precursor to this concept.

In his system, he demonstrated how to generate a **sequence of order 'n'** on an alphabet **A** of size **'k'**, where every possible string of length n appears as a substring exactly once. In Piṅgala's case, the alphabet had two symbols: **Laghu (0)** and **Guru (1)**, and he showed how a sequence could be generated for **n = 3** (a Gaṇa of size 3). This is strikingly similar to the modern **binary De Bruijn sequence**, where you would generate all combinations of binary strings of length 3, ensuring that each combination occurs exactly once.

12.3 Piṅgala's Legacy and Impact on Modern Mathematics

Piṅgala's work is not only significant for its poetic insights but also for the **mathematical concepts** he introduced. His application of binary-like systems, combinatorics, and the formalization of zero (*śūnya*) helped lay the foundations for mathematical fields that would later evolve into disciplines such as **computing, information theory**, and **combinatorial mathematics.**

- **Zero (śūnya):** Piṅgala also formalized the use of the word **śūnya** to denote the concept of **zero**, an idea that would later become fundamental in the development of mathematics.
- **Combinatorics:** His exploration of the ways in which different syllabic combinations can be arranged is an early example of combinatorial principles, which are central to many areas of modern mathematics.
- **Binary Representation:** Piṅgala's binary system (using Laghu and Guru syllables as 0 and 1) is recognized as an early precursor to the modern binary code used in **computing** today.

In summary, Piṅgala's work was an early fusion of **mathematics** and **literature**, and his insights into binary representation and combinatorial mathematics had a profound influence on the development of both fields. His contributions, particularly in linking binary systems to prosody, demonstrate the sophisticated mathematical understanding that existed in ancient Bharat and its far-reaching impact on modern science and technology.

13. Summary

The study of the Bhartiya numeral system, measurement techniques, and their historical significance reveals the profound impact that ancient Bhartiya scholarship has had on mathematics and science. From the development of the decimal system and place-value notation to the invention of zero, Bhartiya mathematicians provided foundational principles that continue to influence modern computational methods. The historical evidence, spanning from ancient texts to archaeological findings, affirms Bharat's pioneering role in shaping global mathematical thought. The integration of mathematics with language, culture, and philosophy in ancient Bharat is another testament to the intellectual ingenuity of early scholars. Systems such as Bhūta-saṃkhyā and Kaṭapayādi exemplify the innovative ways in which numbers were represented, making mathematics more accessible and interconnected with other disciplines. Similarly, the advanced measurement systems used in trade, architecture, and astronomy highlight the practical applications of mathematical knowledge in everyday life.

The concept of zero, one of the most revolutionary contributions, played a crucial role in advancing arithmetic operations, algebra, and even modern computing. The early understanding of combinatorics, binary systems, and place-value notation further illustrates the sophistication of Bhartiya mathematical thought. Piṅgala's work, in particular, foreshadowed principles used in contemporary digital technology, underscoring the timeless relevance of these contributions. Ancient Bhartiya scholars not only developed mathematical concepts but also ensured their application in diverse fields, including astronomy, engineering, and medicine. Their systematic approach to large numbers, precise measurement techniques, and the standardization of units laid the groundwork for scientific advancements that continue to shape the modern world.

In conclusion, the mathematical and scientific heritage of ancient Bharat serves as a cornerstone of modern knowledge. Recognizing these contributions not only honors historical achievements but also inspires future generations to explore the depths of mathematical innovation. By understanding the legacy of Bhartiya mathematics, we gain a deeper appreciation for its role in shaping the evolution of human intellect and scientific progress

Suggestive Readings

1. Rao, Ganti Prasada. *The Indian Number System: At the Center of the Mathematical World*. BS Publications, 2023. ISBN: 9789395038416.
2. Singh, Bal Ram, et al. *Science and Technology in Ancient Indian Texts*. D.K. Printworld (P) Ltd., 2012. ISBN: 9788124606322.
3. Bhanu Murthy, T.S. *Modern Introduction to Ancient Indian Mathematics*. New Age International, 2009. ISBN: 9788122426007.
4. Dahiya, Poonam Dalal. *Ancient and Medieval India*. McGraw Hill Education, 2017. ISBN: 9789352603459.
5. Van Nooten, B. "Binary Numbers in Indian Antiquity." *Journal of Indian Philosophy*, vol. 21, 1993, pp. 31–50. DOI: 10.1007/BF01092744.
6. Chaubey, Pratiksha, Dwivedi, Astha, and Roy, Rajni. "Ancient Indian Time Counting: An Analytical Study." *International Journal for Research in Applied Science and Engineering Technology (IJRASET)*, 2025. ISSN: 2321-9653. DOI: 10.22214/ijraset.2025.67185.
7. Prinsep, James. "British Indian Weights and Measures." In *Essays on Indian Antiquities*, Cambridge University Press, 2013. DOI: 10.1017/CBO9781139507257.011.
8. Gupta, S.V. *Units of Measurement: Past, Present and Future*. Springer, 2012. ISBN: 9783642261534.
9. Desai, D.N. *Indian Mathematics: An Introduction*. Tata McGraw-Hill Education, 2017. ISBN: 9780071329769.
10. Sarma, K.K. *Mathematics in Ancient India*. Springer, 2010. ISBN: 9783642104595.
11. Gupta, M.K. *History of Indian Mathematics*. Cosmo Publications, 2005. ISBN: 9788177555102.
12. Kapoor, S. and R.S. *Mathematical Concepts in Ancient Indian Texts*. Shree Publishers, 2009. ISBN: 9789380027104.
13. Dhavalikar, M.K. "Measurement Systems in Ancient India." *Studies in Ancient Indian Science and Technology*, vol. 12, 1995, pp. 134–140. ISSN: 0970-0857.
14. Katiyar, M. "Exploring the Bhūta-saṃkhyā System in Ancient Indian Mathematics." *Indian Journal of Historical Studies*, vol. 14, no. 2, 2012, pp. 45-62. DOI: 10.1080/21585399.2012.730004.
15. Varma, S. and Shukla, K. *Historical and Conceptual Development of Zero in Indian Mathematics*. Springer, 2018. ISBN: 9789811070586.
16. Sivaramakrishnan, N. *The Concept of Paramāṇu in Vedic Texts and Its Relevance to Modern Physics*. Motilal Banarsidass Publishers, 2010. ISBN: 9788120814103.
17. Narayan, V. *The Scientific Thought of Ancient Indian Mathematicians*. Aryan Books International, 2009. ISBN: 9788187586671.

18. Biswas, S. "Piṅgala's Influence on Binary Code: An Analytical Approach." *International Journal of Modern Mathematical Physics*, vol. 6, no. 4, 2021, pp. 251-263. DOI: 10.1142/S1793962321500299.
19. Sharma, R. *The Mathematical Legacy of Piṅgala and Its Impact on Computer Science*. CRC Press, 2015. ISBN: 9781482250341.
20. Mookerjee, A. *Chandaḥ-śāstra and Its Mathematical Contributions*. Asian Studies Centre, 1996. ISBN: 9780951809302.
21. Agrawal, N. and Patel, M. *Mathematical Insights from Ancient Indian Astronomy*. Springer, 2020. ISBN: 9783030303149.
22. Goswami, K. "Piṅgala's Chandaḥ-śāstra and Binary Numbers." *Mathematics of the Vedas*, vol. 12, no. 1, 2019, pp. 55–72. DOI: 10.1007/s41159-019-00032-2.
23. Jha, R.P. *Time and Measurement in Vedic Literature*. Asian Research Publishing, 2018. ISBN: 9788189671303.
24. Reddy, S. "Piṅgala and the Binary System: Revisiting Ancient Indian Mathematical Thought." *Proceedings of the International Conference on Mathematics*, 2017, pp. 125–136. DOI: 10.1109/ICMATH.2017.174.
25. Bhattacharya, A. *The Mathematical Roots of Ancient Indian Algorithms*. Springer, 2012. ISBN: 9783642262432.
26. Patnaik, A. *Science and Spirituality in the Ancient Indian Texts*. Oxford University Press, 2011. ISBN: 9780195696160.
27. Sharma, P. *The Legacy of Piṅgala: Binary Numbers in Ancient Indian Context*. Cambridge University Press, 2019. ISBN: 9781108471204.
28. Patel, R. "Exploring the Kaṭapayādi System and Its Applications." *Indian Journal of Mathematical History*, vol. 9, no. 4, 2014, pp. 28–39. DOI: 10.1080/21585929.2014.917625.